

THE OPTICAL CHARACTERISTICS OF ABSORBING MEDIA IN THE INFRARED REGION
USING THE MODEL OF A RESONATOR WITH VISCOUS FRICTION

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The classical model of a resonator with viscous friction is used for construction of a corrected resonance curve of continuous bands of strong absorption in crystals. A method of construction of a spectral curve of reflection from three of its values, based on the assumption of correspondence of amplitudes and phases of forced oscillations and reflection, is proposed. The optical constants \underline{n} and \underline{k} are determined graphically from the corrected resonance curve. In the case of continuous bands of strong absorption a combination of the suggested method with the Robinson-Bode method permits making the determination of the phase of the forced oscillations more precise, and also allows for discrepancies caused by schematism of the model used. Comparison of some calculated data with the experimental data for a number of alkali halide crystals showed satisfactory agreement.

Author

1. The present paper is an examination of the infrared spectra of solids having strong continuous bands of absorption using the model of a resonator with viscous friction.

The known solutions of the equation of forced oscillations may be considered as parametric equations of a curve on the amplitude-phase plane. On this plane, each point is characterized (in polar coordinates) by a phase δ and amplitude ρ . Here the frequency serves as a parameter. Eliminating it, as a result of a joint examination of both solutions, one can obtain the expression

$$a = \frac{\Delta a}{2} (\sqrt{\cos^2 \delta + 4Q^2 \sin^2 \delta} + \cos \delta), \quad (1)$$

which is the equation of the corrected resonance curve on the amplitude-phase plane. Here a is the amplitude corresponding to the phase σ (phase displacement between the constraining force and the constrained oscillation), Δa is the amplitude at $\delta = 0$, $Q = \frac{\omega_0}{\gamma}$, where ω_0 is the resonating frequency during the ab-

/Numbers in the margin indicate pagination of the original foreign text.

sence of damping, γ is the coefficient of damping. If $Q \geq 2$ and $\cos^2 \sigma \ll 4Q^2 \sin^2 Q$, then equation (1) for a large part of the curve in a first approximation may be replaced by the equation describing the circumference in the plane under consideration,

$$a \approx a_p \cos(\delta - \delta_p), \quad (2)$$

and where $\operatorname{tg} \delta_p = 2Q$, and δ_p is the phase displacement at a maximal value of the amplitude a_p . The less the damping the closer this resonating phase displacement is to 90° and the smaller the amplitude at a phase displacement equal to zero. In the region of small values of δ , the expression (2) does not describe the phenomenon at any values of Q . At $\delta = 0$ the equation of the circumference (2) gives the value $a(\delta = 0) = \frac{\Delta a}{2}$ instead of the actual value Δa . This permits from the values of the maximal amplitude a_p and the amplitude Δa at $\delta = 0$ ¹⁾ construction of the corrected resonance curve and determination of δ_p and Q (see Figure 1, A). In addition the section AB of the corrected resonance curve is constructed from equation (1). Experience shows that in practice it can be constructed arbitrarily, not introducing noticeable errors.

Suppose that the amplitude and phase of reflection corresponds to the amplitude and phase of the constrained oscillations of a resonator with viscous friction. In this case, knowing the connection of the optical constants with the complex coefficient of reflection $\vec{a} = \rho e^{i\delta}$, one can determine the optical constants by the values of the amplitude and phase of the constrained oscillations of the resonator describing the system. The connection of the optical constants with the complex coefficient of reflection bears a universal character and in no way is connected with the character of the oscillation of the

¹⁾The expression of a_p and Δa from the value of the coefficient of reflection of the dielectric constant and the index of refraction of the material will be given below.

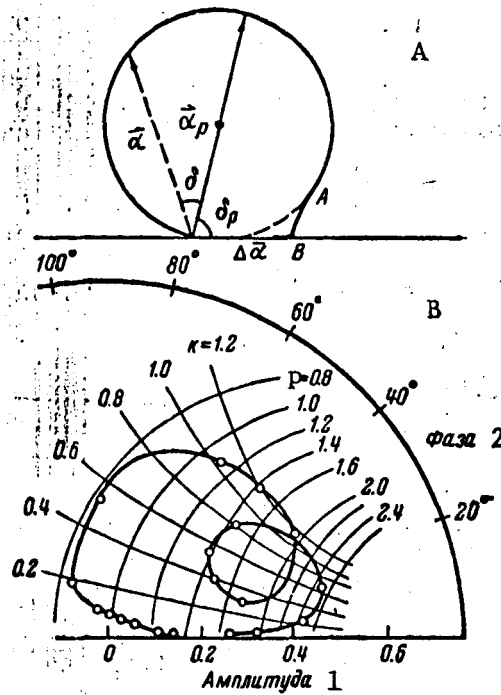


Figure 1. (1) Amplitude; (2) Phase

resonators of the medium. The model of this resonator with viscous friction serves only as an approximation for the construction of a corrected resonance curve in the plane of the complex coefficient of reflection.

In the plane under consideration, the distribution of the complex coefficient of reflection $\vec{a} = \rho e^{i\delta}$ is reflected according to the frequencies for a separate band by a continuous family of vectors, set aside from the origin of the coordinates, the ends of which describe a curve close to the circumference. In the case of the presence of several bands for each value of the frequency the corresponding vectors are added. In Figure 1B, borrowed from the paper of Robinson and Price [1], presents the curve of the complex coefficient of reflection for a band of teflon of about 8.5μ , constructed from the experimental points for each value of the frequency. Having the purpose of determining the optical constants from measurements of the values of the coefficient of reflec-

tion, the author did not pay attention to the correct form of the curves obtained. Figure 1B can be considered as a superposition of two resonance curves described by formula (1). Thus, the results of Robinson and Price's work, to which we shall return below, confirm the applicability of the proposed model for determining the optical curves of substances having strong infrared absorption bands. In Figure 1B two sets of circumferences are also drawn representing a coordinate network with the help of which, it is possible to determine, for each point of the resonance curve, both optical constants. The method of constructing such a network will be described below. /674

The connection of the relative values of the amplitude with the wavelength values can be found exactly (like the equality (1)) from the solution of the equation of the constrained oscillations of a resonator with viscous friction and has the form

$$\frac{a}{a_p} = \frac{1}{\sqrt{1 + F^2(Q) \left[1 - \left(\frac{\lambda_p}{\lambda} \right)^2 \right]^2}}, \quad (3)^2$$

where a is the reflection amplitude at wavelength λ , a_p is the resonance amplitude at wavelength λ_p and $F(Q)$ is determined by the expression

$$\frac{F(Q)}{Q} = \sqrt{\frac{1}{1 - \frac{1}{4Q^2}} - \frac{1}{Q^2}} \rightarrow 1. \quad (4)$$

At $Q \geq 6$, $F(Q) = Q$, correct within 1% (see Figure 2).

²Equation (3) is an exact expression of the corrected resonance curve. Using this expression it is possible to obtain a corrected resonance curve applicable to radio engineering (2),

$$\frac{a}{a_p} = \frac{1}{\sqrt{1 + 4 \left(\frac{\Delta\omega}{\omega} Q \right)^2}}$$

with the assumption $F(Q) = Q$ and $1 - \left(\frac{\omega}{\omega_p} \right)^2 \approx \frac{2\omega\Delta\omega}{\omega^2}$. Moreover, $Q = \frac{\omega_0}{\gamma} = \frac{\omega_0}{\Delta_1\omega}$

($\Delta_1\omega$ is the so-called half thickness) and corresponds to the quality factor (the ratio of the wave resistance to the activity) at the frequency ω_0 .

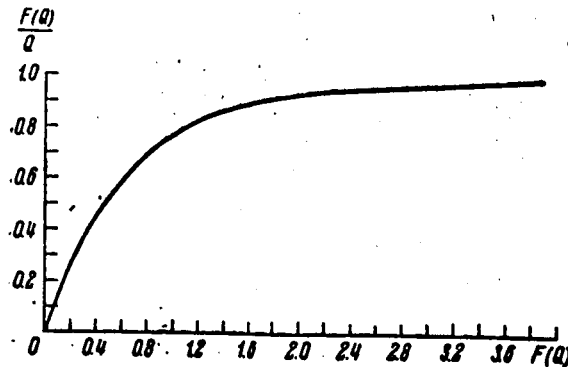


Figure 2.

In practice, however, it is more convenient not to use expression (3) but the parametric expression

$$\operatorname{tg} \beta = \left[1 - \left(\frac{\lambda_p}{\lambda} \right)^2 \right] F(Q),$$

obtained from (3) if you assume $\frac{a}{a_p} = \cos \beta$. Comparison of the latter expression with expression (2) shows that in the region where the resonance curve is sufficiently close to the circumference, $\beta = \delta - \delta_p$. At $\lambda = 0, a = 0$ (the beginning of the curve of Figure 1A) at $\lambda \rightarrow \infty a \rightarrow \Delta a$. Then on the basis of expression (3) we have

$$\Delta a = \frac{a_p}{\sqrt{1 + F^2(Q)}} \quad (5)$$

or solving this equation for a_p and substituting its value in (3) we obtain

$$a = \Delta a \sqrt{\frac{1 + F^2(Q)}{1 + F^2(Q) \left[1 - \left(\frac{\lambda_p}{\lambda} \right)^2 \right]^2}}. \quad (6)$$

2. Using the equations of the first section it is easy to construct, /675 in the plane of the complex coefficient of reflection, the corrected resonance curve (1), to plot on it in accordance with (3) the values of the wavelength, and also, constructing on the same circumference the corresponding network of

the optical constants³ and to obtain their spectral pattern (Figure 1A).

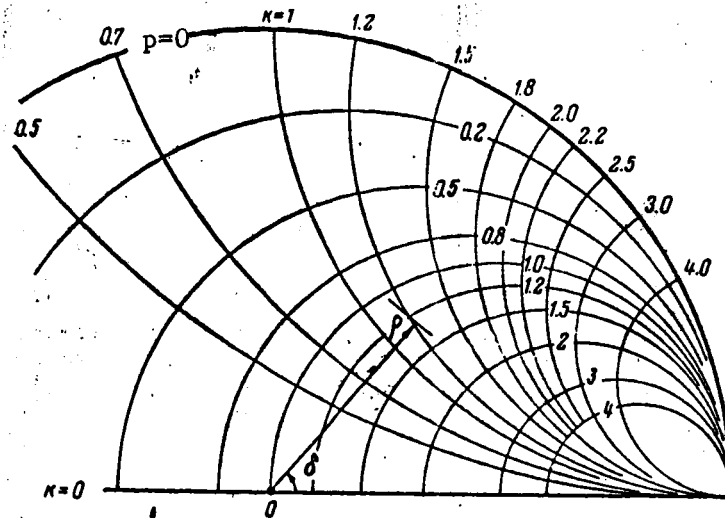


Figure 3.

For construction of the network the expression linking the optical constants of the substance with the complex coefficient of reflection is used in the case of normal slope

$$p_{\text{normal}} = - \frac{n-1-ik}{n+1-ik},$$

or in a different form

$$z' = \frac{z-1}{z+1},$$

where $z' = u + iv$ is the complex coefficient of reflection and $z = x + iy$ is the complex index of refraction.

Expressing u and v by x and y we obtain

$$u = \frac{x^2 + y^2 - 1}{(x+1)^2 + y^2}; \quad v = \frac{2y}{(x+1)^2 + y^2};$$

substituting $y = k$ (constant) and carrying out the conversion, we obtain the equation of the circumference

$$(u-1)^2 + \left(v - \frac{1}{k}\right)^2 = \left(\frac{1}{k}\right)^2$$

³ Such a network was used (without derivation) by a number of investigators (see for example, Male [3] and Robinson [1, 4]). We shall give in later communications a discussion of this graphic connection.

with radius $\frac{1}{k}$ and the center located at the point $u = 1$ and $v = \frac{1}{k}$. Giving k different values we obtain a set of circumferences passing through the point $u = 1, v = 0$, the centers of which lie on the line $u = 1$ (see Figure 3). /676

Similarly, substituting the value $x = p$ we obtain the equation of the circumferences

$$\left(u - \frac{n}{n+1}\right)^2 + v^2 = \left(\frac{1}{n+1}\right)^2$$

with centers on the axis $u = 0$ and radii $\frac{1}{n+1}$, passing through the point $u = 1, v = 0$.

Thus, the components of the complex index of refraction in the plane of the complex coefficient of reflection are represented by two sets of circumferences. At zero absorption ($k = 0$) the circumference corresponds with the infinitely large radius passing through the point $u = 1, v = 0$, that is, the direct, corresponding phase shift at 0 or 180°. The point $\rho = 0$ corresponds to the intersection of a direct transparency and the circumference $p = 1$, that is, with the absence of absorption and an index of refraction equal to unity, the medium will be an optical vacuum -- there will be no reflection. At $\lambda \gg \lambda_p$ where the substance again becomes transparent (implies $k = 0$), the coefficient of reflection $\rho_\infty = \Delta a$. The value of ρ_∞ is linked moreover with the corresponding index of refraction:

If in the substance there are some natural frequencies divided by areas of transparency, then the resonance curve for each of the bands must be constructed from the point p_i on the axis of transparency. Then the coefficient of reflection for any λ in the region of this band is equal to the sum of the vector $\rho = \rho_i = \frac{n_i - 1}{n_i + 1}$, proceeding from the point $\rho = 0$ and going along the axis of transparency to the point p_i and the vector, proceeding from the point p_i , determined by the given band. With long waves much larger than the long-wave

infrared band, it again attains the value $k = 0$, while the coefficient of reflection is constant and is linked with the complete static dielectric constant by the relationship

$$\rho = \rho_{\infty} = \frac{\sqrt{\epsilon} - 1}{\sqrt{\epsilon} + 1}$$

(see Figure 4, giving the normal curves of dispersion and adsorption). Δa (see (5)) in this case is determined by the expression

$$\Delta a = \rho_{\infty} - \rho_0. \quad (7)$$

The presence of natural frequencies in the longer wave regions does not limit the method. Moreover, it is necessary to use the value of the set coefficient of reflection, instead of ρ_{∞} , after the given band. On the other hand, it can be easily shown by geometry that the diameter of the circumference (2) describing the resonance curve is equal to

$$a_p = \frac{2\rho_m^2 - \rho_0^2 - \rho_0\rho_{\infty}}{2\rho_m}, \quad (8)$$

where ρ_m is the maximum amplitude of reflection.

In the case of some overlapping bands it is necessary to construct a number of corrected resonance curves (see below, section 4), and then for the corresponding wavelength to add vectorially the partial values of the complex coefficient of reflection. The corrected resonance curve of the system which is obtained permits the graphic determination of the spectral value of the optical curves.

Therefore, knowing the dielectric constant of the substance, the maximum coefficient of reflection in the given band, the index of refraction in the region of transparency from its shortwave side and the wavelength at the maximum reflection (or at the maximum absorption) one can determine by the graphic /677 method the optical constants in the region of the given band and also the

spectral curve of reflection.

We shall recall briefly the series of operations here:

a) the amplitude of reflection both before and after the given band, and corresponding to ρ_0 and ρ_∞ , is found from the values of the dielectric constant ϵ and the index of refraction ρ_0 (see figure 4);

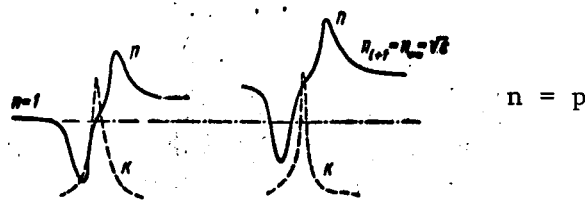


Figure 4.

b) using the value of the amplitude of the maximum coefficient of reflection $\rho_m = \sqrt{R_{\max}}$, ρ_0 and ρ_∞ , the diameter of the circumference a_p using equation (8) and Δa using equation (7) are determined and the resonance curve is constructed (see Figure 5);

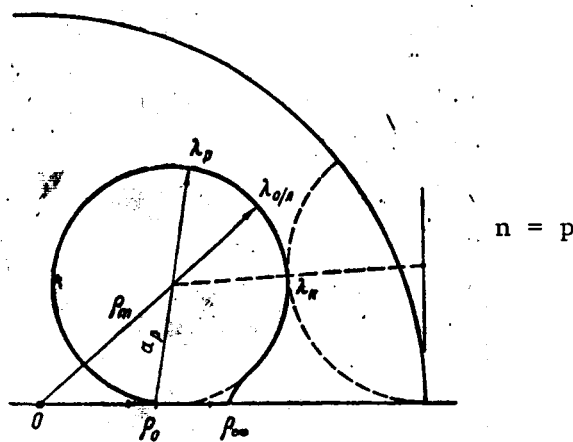


Figure 5.

c) having determined Δa and a_p , $F(Q)$ is calculated using expression (5);

d) having the value of $F(Q)$ from equation (3) (or its equivalent parametric equation) the relative wavelength scale is arranged along the resonance curve;

e) finding the point of contact of the circumference (2) with one of the circumferences $k = \text{const.}$ (see Figure 1B), knowing the wavelength at maximum absorption, the absolute wavelength scale is determined along the resonance curve. The value of the wavelengths at maximum reflection may be used for this purpose, at the same time the corresponding point on the resonance curve is determined from the place where it intersects with the straight line joining the origin of the coordinates ($\rho = 0$) with the center of the circumference (2);

f) the value of ρ is read for each wavelength from the network ρ, δ . The set of the values of $\rho^2 = R$ determines the spectral curve of reflection.

g) from the networks p and k the optical curves are plotted for each wavelength and the appropriate spectral curves are constructed.

If the curve $\rho^2 = R$ differs from the experimental curve then it is necessary to introduce into the values of the complex coefficient of reflection suitable corrections (see section 5) which will enable one both to explain the reasons for these discrepancies and to determine more precisely the optical curves.

3. The graphic method considered above of course only permits an approximate determination of the spectral shape of the optical curves. Moreover, in the region of small absorptions the wavelength scale along the resonance curve becomes very fragmentary, and the curve itself (1) differs significantly from the circumference (2). In addition in the region where the layer of material a few millimeters thick is almost transparent more exact data concerning the indices of refraction is necessary (for example, for the requirements of instru-

ment making). All this compels one to reject approximate graphic methods in this case, especially since for a sufficiently small coefficient of absorption k , the problem can be solved analytically by the exact equation (6). With small k the vector \vec{a} is directed along the axis of transparency with a very large degree of accuracy. Then on the basis of the network p on the plane of the complex coefficient of reflection (Figure 3) $\pm a = \frac{2}{n+1} - \frac{2}{n_0+1}$ (+ with $\lambda < \lambda_p$, - with $\lambda > \lambda_p$) and $\Delta a = \frac{2}{p_0+1} - \frac{2}{n_\infty+1}$. Substituting these values in equation (6) and converting it, we obtain the following expression for $p(\lambda)$

$$p(\lambda) = p_0 - \frac{n_0+1}{1 \pm \frac{n_\infty+1}{n_\infty-n_0} \sqrt{\frac{1+F^2(Q) \left[1 - \left(\frac{\lambda_p}{\lambda}\right)^2\right]^2}{1+F^2(Q)}}}.$$

This expression is true only in the neighborhood of the axis of transparency, but the corrected resonance curve, as we know, approaches the axis of transparency in two regions, in both cases the formula can be considerably simplified.

a) At $2\lambda < \lambda_p$ (the region of transparency before the absorption band) $1 \ll F^2(Q) \left[1 - \left(\frac{\lambda_p}{\lambda}\right)^2\right]^2$, then in the corresponding approximation we obtain with a great degree of accuracy

$$p(\lambda) = p_0 - B \frac{\lambda^2}{C\lambda_0^2 - \lambda^2},$$

where

$$\frac{B}{p_0+1} = \frac{1}{l-1} = \frac{C}{l}; \quad l = \frac{n_\infty+1}{n_\infty-p_0} \frac{F(Q)}{\sqrt{1+F^2(Q)}}. \quad (9)$$

b) At $\lambda > 4\lambda_0$, which corresponds to the region of transparency after the absorption band,

$$p(\lambda) = p_0 + (p_\infty - p_0) \frac{\lambda^2}{\lambda^2 - D\lambda_0^2},$$

where

$$D = \frac{p_\infty+1}{p_0+1} \frac{1}{1 + \frac{1}{F^2(Q)}}. \quad (10)$$

Formulas (9) and (10) express dispersion of the material with the help of those parameters which are used for describing its optical curves in the region of a strong absorption band. In the regions where two bands show up the dispersion the formula takes the form

$$p(\lambda) = 1 + (p_0 - 1) \frac{\lambda^2}{\lambda^2 - D\lambda_p^2} - B \frac{\lambda^2}{C\lambda_p^2 - \lambda^2},$$

where

$$D = \frac{p_0 + 1}{2} \frac{1}{1 + F^2(Q)},$$

and B and C have the same meaning as in formula (9). Generalization of the formula to the case of many bands does not present any difficulty.

4. For illustration of the method, a comparison was made of the calculated data with the experimental data (well-known in the literature) from the optical characteristics of alkali halide crystals and in particular the NaF crystal⁴.

The powerful infrared absorption band in these cases is due to the natural oscillations of the crystal lattice, combined with the relative displacements of the systems of ions of different charge which form the crystal. These oscillations can be roughly considered as natural oscillations of the resonator. Supposing that in this case the amplitude and the phase of the complex coefficient of reflection corresponds to the amplitude and phase of the constrained oscillations in the presence of viscous friction, we shall use our model to determine the optical curves of these substances.

The reflection spectrum of the after rays and the dispersion of the alkali halide crystals we found by the method described above, using the known values of the dielectric constant ϵ , the index of refraction p_0 (see Figure 4), the

⁴A detailed consideration of all alkali halide crystals will be published separately.

wavelengths at maximum absorption and the value of the maximal amplitude of the coefficients of reflection in the region of the "after rays" ρ_m .

The first four columns of Table 1 present the original known experimental data for the NaF crystal, and the following columns given the values of the quantities found using formulas (7), (8), (5) and (9) which enable one to construct the corrected resonance curve and the dispersion formula.

Table 1

Characteristics of the NaF Crystal

$\epsilon[5]$	$p_o[6]$	$\lambda_k[7]$	ρ_m^*	ρ_o	ρ_∞	a_p	$F(Q)$	l	B	C	λ_p	$\sqrt{C\lambda_p}$
6.0	1.3198	40.6	0.97	0.14	0.42	0.845	2.83	2.89	1.227	1.53	32.8	40.6

*The value of ρ_m , equalling from 0.93 to 0.97, can be considered as constant for all alkali halides since the difference of the known values does not exceed the limits of accuracy of the measurement.

It is interesting to note that the quantity $\sqrt{C\lambda_p}$, standing for the characteristic wavelength, coincides with the quantity λ_k for all the crystals examined, which is in good agreement with recent experimental and theoretical data [8, 9]. The values of the index of refraction of sodium fluoride calculated for a number of wavelengths from the data in Table 2 using formula (9) are in good agreement with known experimental data.

Table 2

λ, B, μ	3	4	5	6	7	8	9	10	
$p(\lambda)$	1.3140 1.3131	1.3085 1.3086	1.3018 1.3012	1.2932 1.2928	1.2830 1.2827	1.2707 1.2718	1.2567 1.2572	1.2400 1.2412	Опыт [10] 1 Расчет. 2

(1) Experimental; (2) Calculated

Figure 6 gives the corrected resonance curve specified by the basic

oscillation of the crystal lattice of the NaF crystal. The absolute scale /680 of wavelengths is plotted on the curve on the basis of calculations according to formula (3) and the known value $\lambda_k = 40.6\mu$. The relation $\frac{a}{a_p} = f \left[\frac{\lambda}{\lambda_p}, F(Q) \right]$, described by expression (3), for convenience may be expressed graphically, for example, as a set of curves depending on the parameter $\frac{a}{a_p}$. Using Figure 6, it is possible to determine the value of the wavelength at maximum reflection -- the so-called wavelength of the "after rays" and the wavelength of maximum power transmission in air (the Christiansen-Pfund filter). The maximum transmission of an absorbing powder naturally cannot correspond exactly to the value $p = 1$, and preferably must correspond to the minimum reflection and even must be somewhat displaced from it in the direction of the axis of transparency. Values, found by construction of corrected resonance curves of the type presented in Figure 6, of wavelengths of "after rays" and of wavelengths of maximum transmission of Christiansen-Pfund filters for a number of crystals are compared in Table 3 with known experimental data. The correspondence is proof of the applicability of the model developed to alkali halide crystals of the type under consideration.

Taking the values of the amplitude of the coefficient of reflection for each wavelength from the graph (Figure 6), it is easy to construct a spectral curve of reflection of the NaF crystal (at normal slope). The corresponding curve in Figure 7 is compared with known experimental data [11].

Combining the resonance curve (Figure 6) with a coordinate net of optical constants (Figure 3) one can also determine the value of the optical constants for each wavelength in the absorption band region. Figure 7B illustrates the agreement of the data thus obtained for NaF with known experimental values. In Figure 7A and 7B there is observed some divergence of the cal-

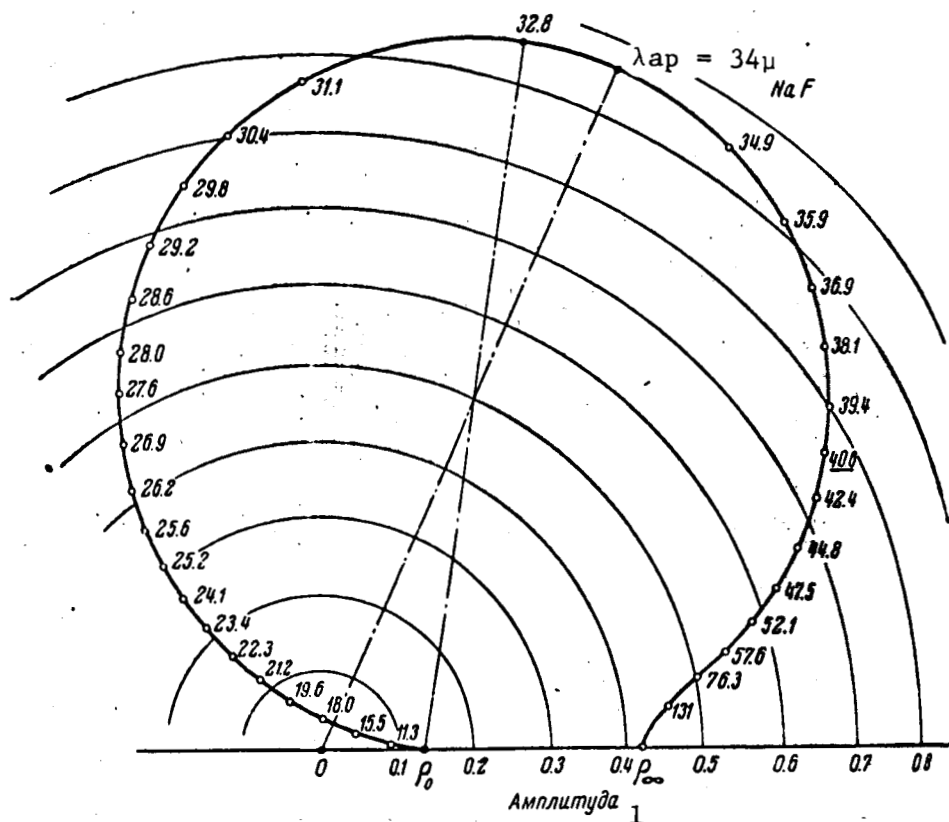


Figure 6. (1) Amplitude

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Table 3

1 Кристалл	LiF	NaF	NaCl	NaBr	NaJ	KCl	KBr	KJ	
2. "After rays" {	26	35	53	—	—	68.4	81.5	94	4 Опыт [11]
лучи λ(μm) . {	26	34	54	65	76	63	81	94	5 Расчет.
3. Фильтр Хри- стиансена — {	11.2	—	32	37	49	37	52	64	4 Опыт [12]
длина . . . {	12	16	33	41	51	40	53	67	5 Расчет.

(1) Crystal; (2) "After rays"; (3) Christiansen-Pfund Filter;
(4) Experimental; (5) Calculated.

culated data from the experimental values which is caused by the presence of a second infrared band in the crystal. Such a band is characteristic of all alkali halide crystals. It is a combined optical-acoustical band, the activity of which in infrared absorption is caused by the appearance of a dynamic dipole

moment due to the deformation of the ions during the basic oscillation of the lattice. Remembering that $Q = \frac{\omega_0}{\gamma} \approx \frac{\omega_p}{\gamma}$, from which the coefficient of damping γ is proportional to $\frac{1}{\lambda_p Q}$, and assuming that γ for the second band is the same as for the first, we obtain $\lambda_1 Q_1 = \lambda_2 Q_2$. Here the indices "1" and "2" designate the first and second bands.

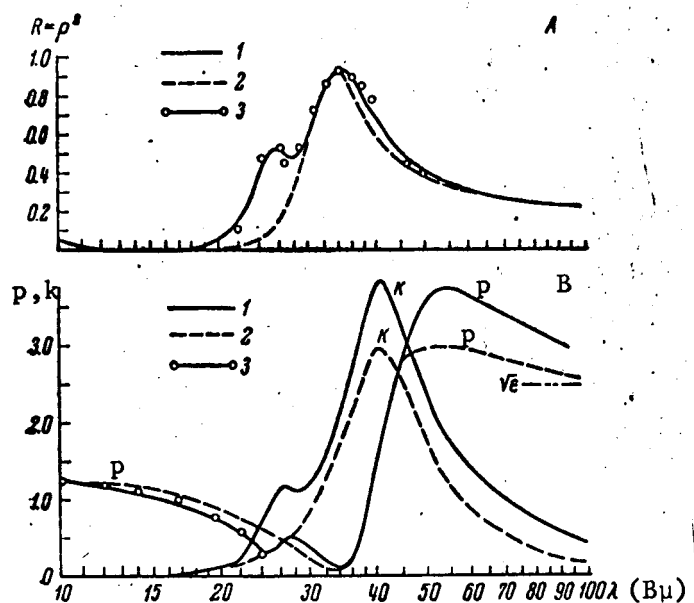


Figure 7.

1- Calculation for two bands, 2 - calculations for first band, 3 - experimental data for A (11) and for B (13).

For sodium fluoride $\lambda_1 = 32.8$, $\lambda_2 = 25\mu$ [11]. On the basis of Table 1 /682 and formula (4) $Q_1 = 2.98$, from which

$$Q_2 = \frac{3.28 \cdot 2.98}{25} = 3.9$$

Therefore, the second band covers the spectral region more narrowly, and being located in the wing of the primary band, has practically no effect on the dispersion (Table 2) and on the position of the powder transmission band (Table 3). The values of the partial reflections of \vec{a} for each wavelength in the pre-

sence of two overlapping bands must be added vectorially. In Figure 8 it is seen that in addition a more complicated pattern is obtained than in the construction of a simple resonance curve (see Figure 1B where the diagram of the complex coefficient of reflection of teflon in the region of two overlapping bands at 8.5μ is presented).

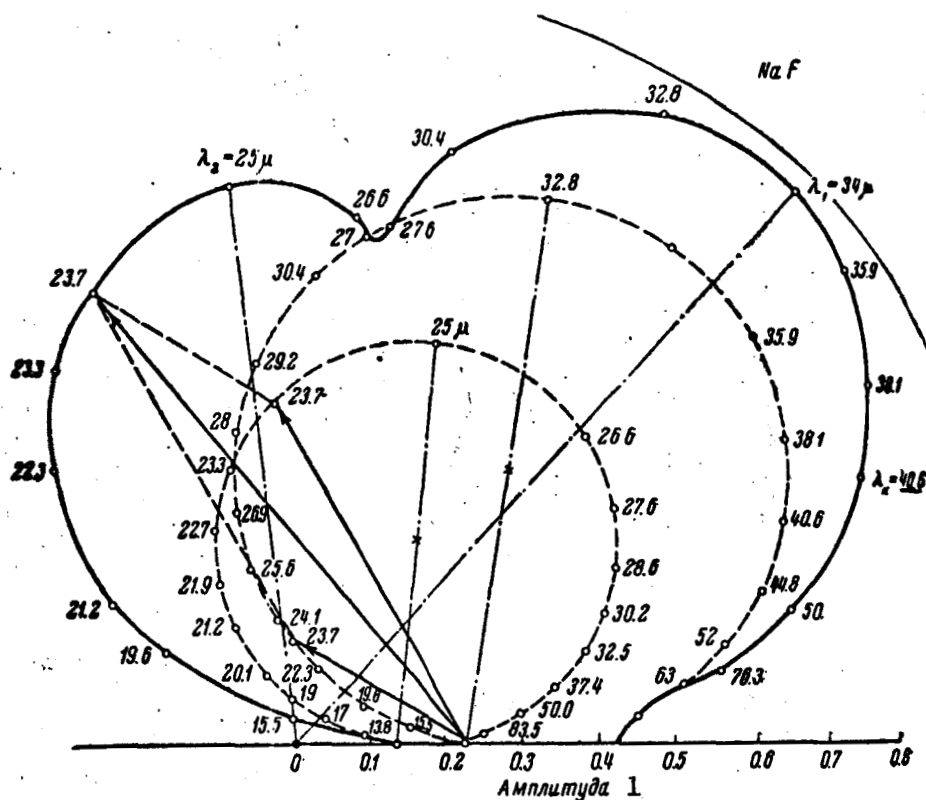


Figure 8. (1) Amplitude

Figure 8 is constructed proceeding from the following assumptions: a) the resonance curve of the primary oscillation varies according to a scale which can be physically substantiated by a decrease in the effective dipole moment taking part in the primary oscillation at the expense of creating dynamic dipole moments; b) the resonance curve of the second band is constructed on the basis of the experimental value of the wavelength of its maximum reflection and

"quality" Q which is found by assuming the constancy of γ .

The scales of the resonance curves are selected so that they satisfy the experimental values of the coefficients of reflection at both maximums.

Figure 7A presents the values of the spectral coefficients of reflection/683 of NaF found using one and two bands, and the experimental data. Completely satisfactory agreement was obtained. Figure 7B presents the values of the optical constants in the region of strong absorption and a comparison with experimental data is made. From this comparison it is seen that the second band has a very small effect on the optical constants even in NaF, in which it is very strongly expressed.

Therefore, according to the data on the index of refraction, the dielectric constant and the wavelength at maximum absorption, it is possible to determine quite exactly the optical characteristics of alkali halide crystals. In 1950 Roberts [14] proposed a method of calculating these characteristics (p_0 , ϵ , λ_k), proceeding from the properties of ions, which allows one, in this way, to predict the optical characteristics of substances not yet obtained. We hope to go into this in detail later.

5. Comparison with experimental data proves the fruitfulness of the method based on the simplest model of a resonator with viscous friction for determining the optical characteristics of ionic crystals in the infrared region of the spectrum. It is necessary to mention that in 1951 Simon [15] attempted to use radio engineering impedance concepts to study the spectral characteristics of quartz in the region of the primary peak of reflection. In addition, he suggested that the oscillation of the "resonators of the medium" are undamped and considered the reflection before and after the band as a background, neutralizing it and subtracting it scalarly from all the values of the coeffi-

cients of reflection. As a result he was forced to use two methods in his papers [15, 16].

The method suggested by us proceeds from a knowledge of three basic parameters which characterize crystals, -- the dielectric constant ϵ , the index of refraction p_0 and the wavelength corresponding to the primary absorption maximum in the infrared region λ_k ; we note that these same parameters are used by Pekar in the polaron theory which he proposed [17]. The method considered is illustrated here for the case of isotropes in an optical relation in the medium, however, it can be expanded to the case of the simplest anisotropes of the medium for which it is sufficient for the coefficient of normal reflection to take the coefficient of normal reflection of plane-polarized radiation, the plane of polarization of which is consequently oriented to the elementary vibrators of the material. Figure 9A, B is confirmation of this, borrowed from the cited work of Robinson and Price [1] in which corrected resonance curves of a monocrystal of urea are plotted in the region of 3μ for light polarized in different planes. Robinson plotted his curves from the points -- for each wavelength the amplitude and phase of reflection was first determined and then the appropriate point was plotted on the amplitude-phase plane and, connecting the set of points, he constructed the curves. The similarity of the shape of his curves is independent verification of the method which we developed in the given case. T. S. Robinson used an extremely intelligent and fruitful method of finding the phase of reflection -- he borrowed the graphical method developed by Bode [18] in the theory of electrical bonds, based on the connection of the complex linear system of phase alteration at a given frequency with the amplitude-frequency characteristic of the system. This method is not directly applicable to strong absorption bands, since with its use very large errors are

gradually accumulated in determining the phase. For cases of strong bands having a structure, a combination of the Robinson-Bode method with the method examined above should be fruitful.

Thus, the suggested method can be used in the general case of complex /684 bands of strong absorption for determining the optical characteristics from the reflection spectrum at normal slope. In addition, from the experimentally determined values of the amplitudes and coefficients of reflection before and after the given band ρ_0 and ρ_∞ and from the value of the amplitude and wavelength at maximum reflection ρ_m and λ_m the approximate values of the spectral distribution of the amplitude and phase of the complex coefficient of reflection are found graphically (by the method under consideration). The difference between the spectral values of the amplitudes of reflection, found by experiment and from the model, as a function of the wavelength, is treated by the Bode-Robinson method [1, 4, 18] finding at each wavelength the additional phase shift. Then from the values of $\rho_0(\lambda) + \Delta\rho(\lambda) = \rho(\lambda)$ and $\delta_0(\lambda) + \Delta\delta(\lambda) = \delta(\lambda)$

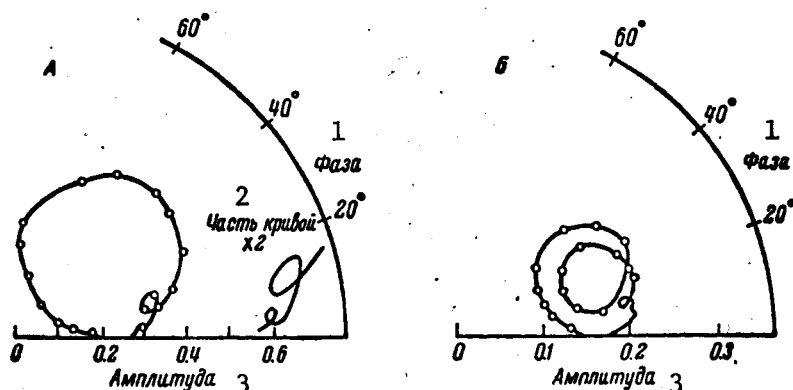


Figure 9.

A -- Electrical vector, parallel to the optical axis; B -- electrical vector, perpendicular to the optical axis. 1) Phase; 2) Part of the curve X 2; 3) Amplitude

the definitive corrected resonance curve of the system is constructed and the spectral values of the optical constants are read from the graph. By this method, on the other hand, the error connected with the determination of phase by the Bode-Robinson method is substantially decreased, and on the other hand possible discrepancies due to the approximate nature of the model used are eliminated.

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